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Parametric Mortality Indexes: From Index Construction to Hedging Strategies

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Life Market

- the market for mortality/longevity-linked securities
- trading/hedging of longevity risk
- lack of liquidity
- creation of mortality indexes

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Mortality Indexes

- model-free indexes
  - Credit Suisse Longevity Index (Credit Suisse 2005), LifeMetrics Index (J.P. Morgan 2007)
  - these indexes are either highly aggregate or specific
  - keep track of a large number of indexes
- model-based indexes
  - stochastic models
  - time-varying parameters

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Model-based Mortality Indexes

- 3 primary criteria by Chan et al. (2014)
  - the new-data-invariant property: to ensure tractability
  - highly interpretable
  - varying age-pattern of mortality improvement
- six stochastic models in Dowd et al. (2010)
  - M1 (Lee-Carter model)
  - M2 (Renshaw-Haberman model)
  - M3 (Age-Period-Cohort model)
  - M5 (Cairns-Blake-Dowd model)
  - M6 (Cairns-Blake-Dowd model with cohort effects)
  - M7 (Cairns-Blake-Dowd model with cohort and quadratic age effects)
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Adapting Mortality Models

- model with age-specific parameters (M1)
  - estimate the model parameters using a restricted sample period $[t_{start}, t_{mid}]$
  - keep the age-specific parameters fixed when we update the model for sample period $[t_{mid}, t_{end}]$
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Adapting Mortality Models

- model with both age-specific and cohort effect parameters (M2 and M3)
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- M1*: adapted M1
  $\ln(m_{x,t}) = \beta_x^{(1)*} + \beta_x^{(2)*}\kappa_t^{(2)}$
- M2*: adapted M2
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Constructing Mortality Indexes

- gender-specific mortality data from 10 populations
  - Australasia: Australia (AUS), New Zealand (NZL)
  - East Asia: Taiwan (TWN), Japan (JPN)
  - Nordic region: Norway (NOR), Sweden (SWE)
  - Western Europe: England and Wales (EW), France (FRA)
  - North America: Canada (CAN), United States (USA)
- data source: Human Mortality Database
- age range: 40-90
- data sample period for NZL: [1950,1993], [1994,2008]
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- maximum likelihood estimation (MLE)
- model selection criterion
  - reduction in log-likelihoods between original model and adapted model
  - Bayesian Information Criterion (BIC)
- M7* gives the best BIC values and the smallest reductions in log-likelihood values
- construct mortality indexes using $\kappa_t^{(1)}, \kappa_t^{(2)}, \kappa_t^{(3)}$ in M7*

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Constructing Mortality Indexes

	Males					Females				
Pop	M1*	M2*	M3*	M6*	M7*	M1*	M2*	M3*	M6*	M7*
AUS	3734	3125	1624	335	19	1045	6249	1431	1150	24
CAN	6948	24836	1404	203	15	1060	6809	805	1313	194
EW	13886	2362	2600	90	83	9566	1633	2353	1252	27
FRA	2051	2282	3466	4539	1820	7847	10668	4107	14868	545
JPN	24212	1805	4118	2655	537	88416	2707	3853	13185	261
NZL	972	830	399	32	1	555	538	389	191	38
NOR	359	2351	437	54	4	94	267	377	480	27
SWE	979	364	902	190	12	361	206	892	943	76
TWN	8033	6151	951	910	5	3415	1886	718	505	15
USA	53358	14229	10554	8860	1766	20218	11028	7202	14371	2224

Constructing Mortality Indexes

	Males						Females					
Pop	M1*	M2*	M3*	M5	M6*	M7*	M1*	M2*	M3*	M5	M6*	M7*
AUS	15766	14937	11419	11942	8894	8199	10051	20938	10810	17545	10452	7965
CAN	22776	58868	11543	10697	9298	8781	10677	22564	10066	17673	11453	8752
EW	39538	14465	14497	16137	9887	9454	30549	12765	13763	24662	12298	9097
FRA	15899	14421	16359	56935	23612	13372	26739	30858	17351	121168	41037	11041
JPN	62618	14144	19169	40456	19438	12627	187997	15358	17880	132618	38947	12164
NZL	8852	9100	7752	6833	6902	6848	7880	8375	7605	7869	7213	6801
NOR	7801	12354	7989	7338	7057	7021	7061	7922	7621	9977	7716	6849
SWE	9668	8935	9499	9175	8035	7652	8269	8460	9273	14262	9405	7589
TWN	24720	21321	10446	15448	10333	8580	14829	12271	9449	12044	8820	7888
USA	123233	40388	34770	102543	40562	17818	54680	33198	25935	122063	52332	16553

K-forward

- standardized mortality-linked security
- a swap between a fixed amount (pre-determined forward value) and a random amount (realized index value) related to one of the three indexes in a reference year
- K1-forward, K2-forward, K3-forward

$$Y \times (\tilde{\kappa}^{(i)}_{t^*} - \kappa^{(i)}_{t^*}), \quad i = 1, 2, 3.$$

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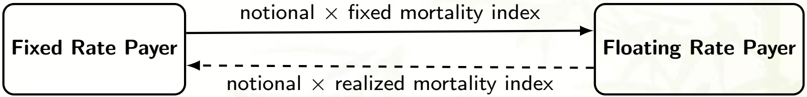

$$Y \times (\tilde{\kappa}_{t^*}^{(i)} - \kappa_{t^*}^{(i)}), \quad i = 1, 2, 3.$$

Key K-duration (KKD)

- similar to key q-duration (Li and Luo, 2012) for q-forwards
- 'key': K-forwards are only available in certain key years $t_1, t_2, \ldots, t_n$
- measures the change in the value of a liability with respect to a small change in a key K-index
- two assumptions
  - a shock in $\kappa_{t_j}^{(i)}$ is accompanied by a level shift in $\kappa_t^{(i)}$ over the period of $t_j \leq t < t_{j+1}$
  - the shock on $\kappa_t^{(i)}$ has no impact on $\kappa_t^{(h)}$ for all $i \neq h$ and t
- $KKD_i(P(\boldsymbol{\kappa}), j) = \frac{\partial P(\boldsymbol{\kappa})}{\partial \kappa_{t_j}^{(i)}}$

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Building Longevity Hedge

- KKD strategy
- KKD of liability portfolio is estimated numerically
- KKD of K-forward can be derived analytically
- KKD of liability portfolio = KKD of hedge portfolio consisting of K-forwards, for each key K-index in each key year
- determine the required notional amounts of K1-forward, K2-forward and K3-forward separately in respective key years

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Hedging Illustrations: Single Cohort

- pension plan coverage: $1 at the beginning of each year from age 65 until the pensioner dies or attains age 91
- mortality data: English and Welsh males, ages 40-90, [1950,2009]
- reference years: 2015, 2020, 2025, 2030
- interest rate: 3% flat
- parametric bootstrap (see Brouhns et al., 2005) simulation: 5000 scenarios
- amount of longevity risk reduction:
  $R = 1 - \frac{\text{variance of PV of unexpected cash flows after hedging}}{\text{variance of PV of unexpected cash flows without hedging}}$
- simulation models: M7*, M5, M3, M2, MRW (Bell, 1997)

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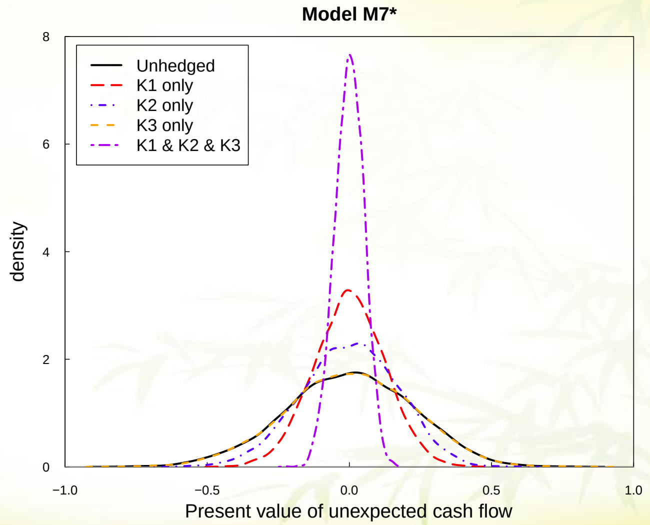
Hedging Illustrations: Single Cohort

- KKD strategy: simple calibration
- optimal hedge: simulations + numerical optimization

Simulation model	KKD strategy	Optimal hedge
M7*	94.7%	97.3%
M5	96.0%	99.1%
M3	95.6%	96.6%
M2	95.2%	95.8%
MRW	93.5%	94.0%

Hedging Illustrations: Single Cohort



Hedging Illustrations: Single Cohort

- Sampling risk (small-sample risk): smaller R for smaller number of pensioners
- Sensitivity tests
  - interest rate: R is not sensitive to the interest rate assumption
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  - age range: R of K1- and K3-forward increase for an older age range, but R of K2-forward drops
- advanced ages: satisfactory R for pension coverage until age 101

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Hedging Illustrations: Multiple Cohorts

- consider a multi-cohort pension plan with a coverage from age 60 to 91
- with both active members (ages 50-59) and retirement pensioners (ages 60-90)
- compare K-forward hedge with q-forward hedge
- K-forward: reference year
- q-forward: reference age and reference year
- K-forward hedge is easier to calibrate using key K-index
- q-forward hedge requires key cohorts and key q-rates

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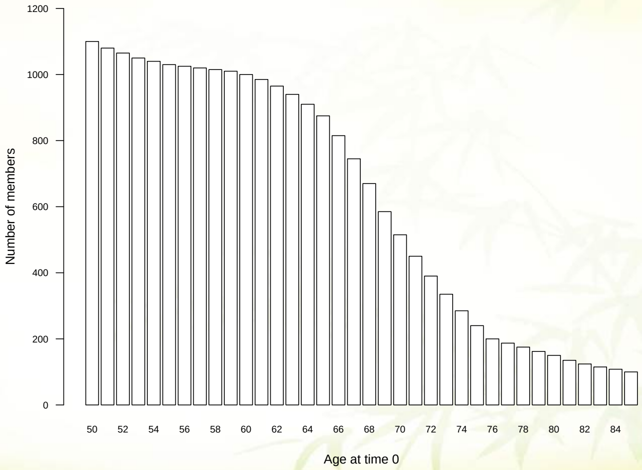

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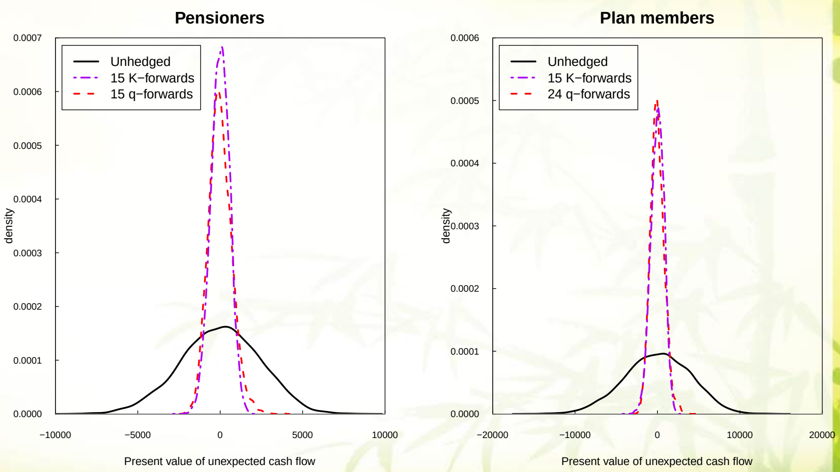

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- the number of instruments required to produce a satisfactory hedge using K-forward remains the same, but that of q-forward rises with a larger number of cohorts
- due to the reference rates of K-forward and q-forward contracts
- K-forwards are potentially more liquid

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- due to the reference rates of K-forward and q-forward contracts
- K-forwards are potentially more liquid

Concluding Remarks

- adapting mortality models to achieve the new-data-invariant property
- constructing mortality indexes using model M7*
- securitization of K-forward
- KKD hedging strategy yields excellent longevity risk reduction
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Future Research

- convexity measure in calibrating the hedge
- dynamic hedging
- population basis risk

Q&A

Thank you!